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Enhancement of the Anomalous-Skin-Effect Fields beneath a Rough Surface and Its Effect on the Correlation-Produced Magnetoplasma Peak

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The surface-roughness model, which describes the surface impedance of potassium as a function of the strength of the magnetic field normal to the surface, is used here to calculate the deep-field enhancement factor – i.e., the amount by which the field deep in the bulk of a rough-surfaced sample exceeds that in the bulk of a diffuse-surfaced sample. Having obtained this factor, we recalculate the change in surface impedance caused by Fermi-liquid effects, using the enhanced field in the calculation rather than the original diffuse field. This procedure should yield the change in surface impedance caused by correlations among the conduction electrons in a sample having a rough surface. We find that the peak in surface resistance calculated in this way is fully comparable in size with what has been observed, and that its location is very close to $\omega_c/\omega = (1 + A_2)^{-1}$, where A_2 is the second Landau-theory Fermi-liquid parameter. The value of A_2 deduced in this way is -0.03, in agreement with the value deduced by Platzman, Walsh, and Fòo from their theory and observation of the high-frequency waves.

I. INTRODUCTION

Recent measurements of the surface resistance of potassium in the anomalous-skin-effect regime[1,2] exhibit a dependence on the strength of the magnetic field which can only be understood on the assumption that the mean free path for electrons close to the surface is much shorter than the mean free path for electrons deeper in the bulk of the sample.[3] They also exhibit a resonancelike anomaly which has been ascribed[1,4] to excitation of the correlation-produced magnetoplasma mode.[5] Attempts to calculate the existence, size, and magnetic field dependence of this anomaly have met with only partial success: If one assumes specular scattering for electrons at the surface, the calculated size of the anomaly is about 10^{-4} of what is observed.[1] If one assumes diffuse scattering, the calculated size rises to about 10^{-2} of what is observed, but the anomaly seems to have the wrong phase.[4] In this paper, we shall show that the same surface-roughness model,[3] and approximately the same parameters which are needed to explain the magnetic field dependence of the surface resis-

tance, also account for the observed size of the anomaly and one of the characteristic features of its shape – a positive peak very close to $\omega_c/\omega = (1+A_2)^{-1}$. (The A_l are the dimensionless parameters of the quasiparticle interaction function in the Landau theory of Fermi liquids. ω_c is the cyclotron frequency of the conduction electrons, and ω is the frequency of the circularly polarized wave.) We do this by using the surface-roughness model of Ref. 3 to calculate a deep-field enhancement factor. This factor is defined as being the ratio of the field deep within a rough-surfaced sample to the field at the same depth within a sample whose surface is diffuse. Having obtained this factor, we recalculate the change in surface impedance caused by Fermi-liquid effects using the enhanced field, rather than the field in a diffuse-surfaced sample. The result should give the change in surface impedance caused by correlations among the conduction electrons in a sample whose surface is rough. If we accept that this is so, then the above results follow by carrying out the calculation just described.

In Sec. II, we carry out the first half of this program, using the model of Ref. 3 to calculate the field enhancement factor. In Sec. III, we evaluate the resulting expressions for a range of parameters relevant to the experiments. Finally, in Sec. IV, we use the enhancement factor to recalculate the surface impedance change caused by the Fermi-liquid effects, and we compare the result to experiment.

II. FIELD ENHANCEMENT FACTOR

In this section, we study the equation governing the field in a rough-surfaced sample and recast that equation in a form which facilitates comparison with the field in a diffuse-surfaced sample, using the model proposed in Ref. 3. The essence of that model is that a sample with a rough surface is simulated by letting the electron mean free path depend on the distance of the electron from the surface. The mean free path is shortest at the surface of the sample ($z=0$), and then increases with distance from the surface until at some depth d (the thickness of the roughened region), the mean free path becomes and remains equal to l_B, the bulk value:

$$l(z) = l_B, \quad z \geq d \ , \tag{2.1a}$$

$$l(z) < l_B, \quad 0 < z < d \ . \tag{2.1b}$$

[In Ref. 3, a particularly simple form of $l(z)$ was used, namely, $l(z<d)=\text{const}$. We shall use that same simple form for evaluation here although it is clear that more realistic choices can easily be used.] Those electrons which do penetrate the region of augmented scattering and strike the surface are assumed to suffer diffuse reflection.

The consequence of this assumption is that a transverse, circularly polarized electric field in the sample will satisfy the nonlocal wave equation

$$\left(\frac{d^2}{dz^2} + k_0^2\right) e(z) + i\omega\mu_0 \int_0^\infty \sigma(z, z') e(z')\, dz' = 0 \ , \tag{2.2a}$$

$$e(z) \to 0 \quad \text{as} \quad z \to \infty \ , \tag{2.2b}$$

and a normalization condition which, for convenience, we take as

$$e(z=0) = 1 \ . \tag{2.2c}$$

σ, the nonlocal conductivity, is calculated by solving the linearized Boltzmann equation. In the presence of a magnetic field normal to the surface, that solution gives

$$\sigma(z, z') = \frac{3nq^2}{4p_F} \int_0^1 u^{-1}(1-u)\, du\, e^{-l(z,z')/u} \ , \quad z > z' \tag{2.3a}$$

$$\sigma(z, z') = \sigma(z'; z) \ , \quad z' > z \ , \tag{2.3b}$$

where

$$l(z, z') \equiv \int_{z'}^{z} dy \left[l^{-1}(y) - i(\omega \pm \omega_c)/V_F\right] \ . \tag{2.4}$$

The $\pm$ sign refers to the two senses of circular polarization. $k_0 = \omega/c$, n is the density of conduction electrons, p_F is their Fermi momentum, and q is their charge. μ_0 is the permeability of free space.

We are to compare this field with that in the diffuse slab – i.e., where there is no added surface scattering. In the diffuse case, the field $e_d(z)$ satisfies

$$\left(\frac{d^2}{dz^2} + k_0^2\right) e_d(z) + i\omega\mu_0 \int_0^\infty \sigma_0(z - z') e_d(z')\, dz' = 0 \ , \tag{2.5a}$$

$$e_d(z) \to 0 \quad \text{as} \quad z \to \infty \ , \tag{2.5b}$$

$$e_d(z=0) = 1 \ . \tag{2.5c}$$

The conductivity σ_0 is calculated using (2.3) and (2.4) but with $l(y)$ replaced by l_B. From this and (2.1), it follows that

$$\sigma(z, z') = \sigma_0(z - z') \ \text{ if } \ z > d \ \text{and} \ z' > d \ . \tag{2.6}$$

In order to compare the two fields, we introduce $G(z, z_0)$, a Green's function for the diffuse equation:

$$\left(\frac{d^2}{dz^2} + k_0^2\right) G(z, z_0) + i\omega\mu_0 \int_0^\infty \sigma_0(z - z') G(z', z_0)\, dz' = \delta(z - z_0) \ , \tag{2.7a}$$

$$G(z, z_0) \to 0 \quad \text{as} \quad z \to \infty \ , \tag{2.7b}$$

$$G(0, z_0) = 0 \ . \tag{2.7c}$$

Use of this Green's function allows us to rewrite

(2.2) as an integral equation

$$e(z)=e_d(z)-i\omega\mu_0\int_0^\infty dz_1\,dz_2\,G(z,z_1)\times[\sigma(z_1,z_2)-\sigma_0(z_1,z_2)]e(z_2) \quad (2.8)$$

or equivalently,

$$e(z)=f(z)e_d(z)\ , \quad (2.9)$$

where

$$f(z)=1-ip\int_0^\infty dz_1\int_0^\infty dz_2\times F(z,z_1)V(z_1,z_2)e(z_2)\ , \quad (2.10)$$

$$F(z,z_1)\equiv G(z,z_1)/e_d(z)\ , \quad (2.11a)$$

$$p\equiv 3\pi\omega\mu_0 nq^2/4p_F\ , \quad (2.11b)$$

$$V(z_1,z_2)\equiv(1/\pi)\int_0^1 u^{-1}(1-u^2)\,du\times(e^{-l(z,z')/u}-e^{-a(z-z')/u})\ , \quad (2.11c)$$

and

$$a\equiv l_B^{-1}-i(\omega\pm\omega_c)/V_F\ . \quad (2.11d)$$

We have shown elsewhere[6] that

$$G(z,z_1)=-\int_0^{z_1} e_d(u)e_d(u+z-z')\,du\ , \quad (2.12)$$

where $e_d\equiv 0$ for $z<0$. Taking $z-z_1$ to be much larger than δ, the anomalous skin depth, we can expand $e_d(u+z-z')$ as a power series in u and then evaluate (2.12). The zeroth-order term is then accurate to order $\delta/(z-z_1)$ or $z_1/(z-z_1)$, whichever is better. From this zero-order term, we get

$$F(z,z_1)=\frac{-e_d(z-z_1)}{e_d(z)}\int_0^{z_1} e_d(u)\,du\ . \quad (2.13)$$

At large values of az, it is known that $e_d(z)$ behaves like $(az)^{-2}e^{-az}$. Hence, for $|az|\gg 1$, we have

$$F(z,z_1)=-e^{az_1}\int_0^{z_1} e_d(u)\,du\ . \quad (2.14)$$

Inserting this result back into (2.10) shows that for $az\gg 1$, $f(z)$ becomes independent of position. The limiting value, which we denote simply as f, is

$$f=1+ip\int_0^\infty dz_1\,dz_2\int_0^{z_1} du\,e^{az_1}e_d(u)V(z_1,z_2)e(z_2)\ . \quad (2.15)$$

This quantity f, so defined, is the field enhancement factor described in Sec. I, because (2.9) gives us

$$e(z)=fe_d(z),\quad |az|\gg 1\ .$$

Although the value of f seems to depend on knowing both $e(z)$ and $e_d(z)$, it will turn out that for $d\gg\delta$, knowledge of $e_d(z)$ suffices. To see why, note that because of (2.1), V vanishes when z_1 and z_2 both exceed d, the thickness of the roughened region. We can, therefore, break the integral in (2.15) into two parts,

$$\int_0^\infty\int_0^\infty dz_1\,dz_2=\int_0^\infty dz_1\int_0^d dz_2+\int_0^d dz_1\int_d^\infty dz_2\ . \quad (2.16)$$

The field $e(z_2)$, which appears in the integrand, will, like the diffuse field, give rise to shielding currents which create a skin effect – i.e., a relatively thin region near the surface within which the amplitude of the field decays rapidly. If the thickness d is greater than the thickness of the skin region, then the second integral on the right-hand side of (2.16) will be negligible compared with the first integral on the right-hand side, and we can drop it. We are then left with a need to know the field $e(z)$ *only* in the roughened region $0<z<d$.

Recall now that in the anomalous-skin-effect limit, the electric field near the surface of a diffuse-surface slab or a specular-surface slab is independent of the mean free path[7,8] – basically because resonant collisionless damping is the primary loss mechanism in this regime. Although this result follows from the mathematical analysis of a situation in which the mean free path is independent of position, it is hard to believe that allowing the mean free path to become position dependent, as we are doing here, could alter this situation. Note that we are *not* making this claim for the deep field, which *does* depend on the mean free path in the homogeneous medium; we are asserting this only for that field near the surface which extends for the order of a skin depth. We therefore assert that it is valid to replace $e(z_2)$ in the first term on the right-hand side of (2.16) by $e_d(z)$, the field in the diffuse slab. These replacements convert (2.15) to

$$f=1+ip\int_0^\infty dz_1\int_0^{z_1} du\int_0^d dz_2\,e^{az_1}e_d(u)V(z_1,z_2)e_d(z_2)\ , \quad (2.17)$$

Note, however, that d here must not be so large that the small difference between $e_d(z_2)$, which extends over a great depth, can contribute to the integral. We have in mind that d will be on the order of, but larger than, a skin depth.

Finally, recall that the field $e_d(z)$ is independent of magnetic field to the same extent (and for the same reason) that it is independent of mean free path. For simplicity then, we can replace $e_d(z)$ by its functional form at cyclotron resonance. This is known to be a function of $W=sz$, where $s=p^{1/3}$. That is,

$$e_d(z)=E(sz)=E(w)\ . \quad (2.18)$$

[A graphic representation of $E(w)$ appears as Fig. 4.5 in an article on transport properties of electrons in metals, by Chambers.[7]] This casts (2.17) into the form

$$f=1+i\int_0^\infty dy_1\int_0^{y_1} dt\int_0^{sd} dy_2\times E(t)e^{ay_1/s}V(y_1/s,y_2/s)E(y_2)\ , \quad (2.19)$$

$$s=(3\pi\omega\omega_p^2/4V_Fc^2)^{1/3}\ .$$

This integral is our expression for the field enhancement factor. Before evaluating it, which we do in Sec. III, let us restate those approximations made in obtaining it, in order that the conditions for which it is expected to be valid are clearly understood. (a) Anomalous-skin-effect conditions hold for the bulk parameters, i.e., $s/a \gg 1$. (b) Anomalous-skin-effect conditions hold for the surface parameters as well, i.e., where $l(z=0)$ replaces l_B. (c) The thickness d of the roughened region is greater than the anomalous-skin depth but less than that required for the deep-field region, i.e., $d|a| \ll 1 \ll ds$.

III. EVALUATION FOR SIMPLEST MODEL

The simplest choice of mean free path consistent with (2.1) is to assume a constant amount of added scattering through the entire roughened region, i.e., to take

$$l^{-1}(z) = l_B^{-1}, \quad z \geq d \ , \tag{3.1a}$$

$$l^{-1}(z) = l_B^{-1} + l_s^{-1}, \quad z < d \ . \tag{3.1b}$$

This was the specific form used in Ref. 3. Crude, as this choice is, it has the virtue of being able to exhibit the sort of effects to be expected with the greatest possible economy of added parameters. We need only introduce two parameters l_s (a surface mean free path) and d (a surface thickness) to describe the roughened region.

It is clear from (2.11c), (2.19), and (3.1) that the parameters on which f depends are sl_B, sl_s, sd, and $(\omega \mp \omega_c)/V_F s$. In Figs. 1–3 we plot the magnitude and phase of f versus $(\omega - \omega_c)/V_F s$ in the neighborhood of cyclotron resonance. In Fig. 1, the plots are made for a range of surface mean free paths, holding the bulk mean free path and surface thickness constant. In Figs. 2 and 3, each of the other two parameters is varied. [The preparation of these figures involves one further numerical approximation which is to be regarded as a computational convenience. Instead of using the function $E(w)$, which is given graphically in Chambers's article,[7] we have a numerical representation, of the form

$$E(w) \approx Ae^{irw} + (1-A)e^{iqw} \ . \tag{3.2}$$

This is a form which was used in Ref. 3, where the parameters A, r, and q were determined variationally for a rough surface. Here, we have repeated that variational procedure using the conductivity σ_0 for a diffuse surface, and have obtained the values

$$A = 2.42 + 0.043i \ , \quad r = 0.415 + 0.712i \ ,$$

$$q = 0.247 + 0442i \ . \tag{3.3}$$

One could undoubtedly adjust these values to obtain a better fit to Chambers's graphs, but the above values yield a fit which is good enough for our use here, where such a crude choice of $l(z)$ is involved. In Fig. 4, we have plotted (3.2) and have superposed values taken from Chambers's graphs to facilitate comparison.]

Use of (3.1) and (3.2) in (2.19) allows one to carry out all integrals analytically, the result being an expression containing logarithms and exponential integral functions of the various complex arguments. The final expression has too many terms to be useful analytically. The easiest way to see what it means is to evaluate it numerically, which is simple to do, and to present the results as we have done.

IV. EFFECT ON CORRELATION-PRODUCED MAGNETOPLASMA PEAK

The presence of short-range forces leads to correlations among the electrons and these can, in turn, alter the surface impedance of a metallic sample from what would be expected in their absence. It has been possible to calculate the change in the surface impedance caused by correlations, assuming that the surface is diffuse.[4] That calculation made it plausible that any multiplicative change in the field which exists in the metal (ignoring correlations to calculate this field) will alter the change in the surface impedance, enhancing that change by the *square* of the multiplicative factor.

Let us recall the argument of Ref. 4 which made this conjecture plausible. The calculation of surface impedance in Ref. 4 was a variational calculation. It utilized a trial field whose form was the sum of the correlationless field (i.e., the electric field which would be present in the metal were there no correlation effect) and two plane waves, one of which had a propagation constant like those in the anomalous-skin-depth field, and the other of which had a propagation constant on the order of that of the correlation-induced mode. The amplitudes and propagation constants of the plane waves were to be adjusted variationally by requiring that a particular functional (in this case, the surface admittance) be stationary. After a considerable amount of analysis, we were able to show that the part of the functional which gives rise to the peak in the surface impedance behaves essentially like the square of integral of the product of the correlationless field and the plane wave whose propagation constant is of the order of that of the mode.

The appearance of such an integral is to be expected wherever the coupling between two waves is a bulk phenomenon (i.e., where the interaction

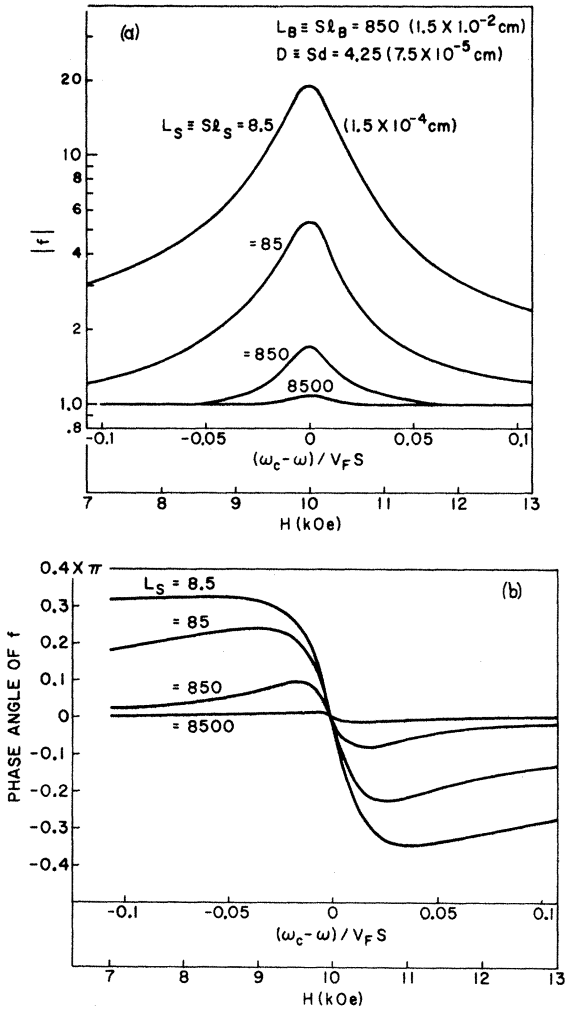

FIG. 1. Magnitude (a) and phase (b) of the field enhancement factor f as a function of magnetic field for various values of the surface mean free path. The curves are labeled both by dimensionless variables and by the dimensional variables which would describe an experiment in potassium at a frequency $\omega = 1.45 \times 10^{11}$ sec^{-1}. Potassium is described by a conduction-electron plasma frequency $\omega_p = 6.07 \times 10^{15}$ sec^{-1} and a Fermi velocity $V_F = 7.1 \times 10^7$ cm sec^{-1}. Cyclotron resonance (at this frequency) is taken to occur at 10 kOe.

is spatially uniform, that is to say, is characterized by $k = 0$). In such a situation, the mode responds to the component of the field whose wavelength matches that of the mode, and just such an integral as we have described gives that component. The strength of the coupling – the constant multiplying the integral – is, in this case, $C_2/\omega\tau$, where C_2 is $A_2/(1+A_2)$, A_2 being the Fermi-liquid theory interaction parameter. That the size of the peak involves the square of the integral is something which we would not have guessed and for which we have no physical explanation. It suggests that, had we known how to do the calculation by perturbation theory, we would have had to go to second order to find the peak. At any rate, the presence of this integral implies that the interaction takes place in the bulk, rather than at the surface, in the sense that the mode responds to a long-wavelength component of the driving field. We should then expect the driving field in the bulk, rather than at the surface, to be the main determinant of the strength of the mode.

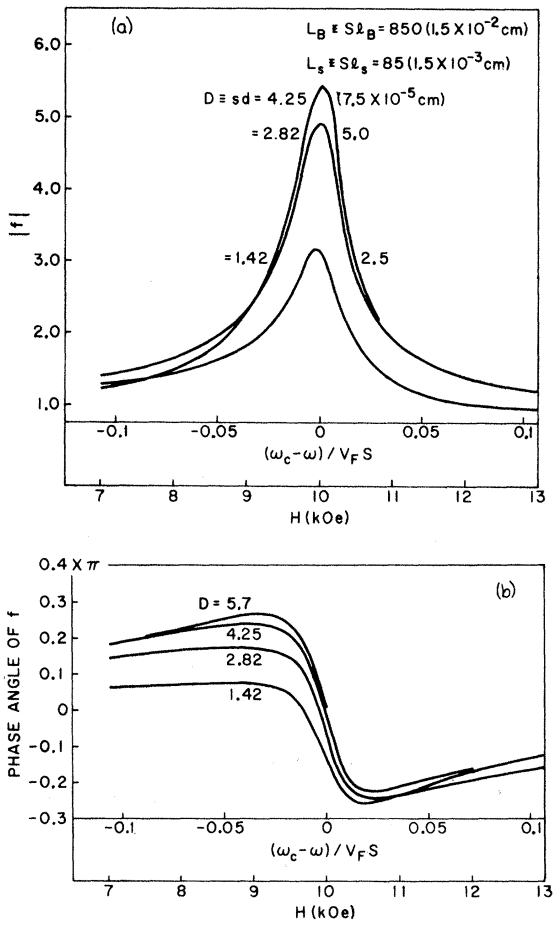

FIG. 2. Magnitude (a) and phase (b) of the field enhancement factor f as a function of magnetic field for various values of the surface thickness. Again, curves are labeled both by dimensionless and dimensional variables, the dimensional variables again referring to potassium at $\omega = 1.45 \times 10^{11}$ sec^{-1}. The tendency for f to become independent of surface thickness when the thickness is somewhat greater than the penetration depth of the fields is evident in (b) and even more so in (a) where we omitted plotting the fourth curve ($D = 5.7$) because it falls so closely on the third curve ($D = 4.25$).

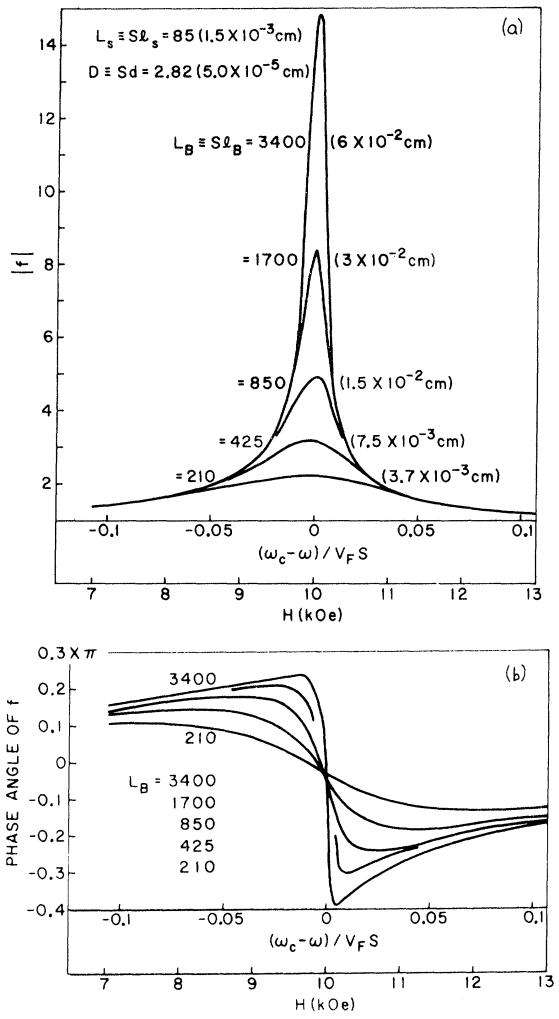

FIG. 3. Magnitude (a) and phase (b) of f for various values of the bulk mean free path.

In the case of a spatially dependent coupling, instead of the integral times the coupling constant, we would expect to find

$$\int_0^\infty dz\,\Phi_0(z)H_1(z)\times(\text{plane wave}) \ ,$$

where Φ_0, as before, is the correlationless field, and $H_1(z)$ is the coupling constant, now dependent on position. Here, the mode responds to the long-range part of $\Phi_0\times H_1$, implying that the value of the coupling constant *in the bulk*, rather than at the surface, is important. This is unchanged in our surface-roughness model.

This reasoning here is conjectural, of course, but partial supporting evidence is provided, as we have noted in Ref. 4, by the fact that the relative sizes of the peaks for a specular surface and a diffuse surface (both of which can be calculated) are in accord with what the above argument would have predicted.

The field enhancement factor which we introduced in Sec. II is not, strictly speaking, a multiplicative factor: It does depend on position. It does become a multiplicative factor (i.e., it loses its depth dependence) when $|az| \gg 1$. Suppose, temporarily, that f were strictly independent of position. Then the field below a rough surface would be a numerical multiple of that beneath the diffuse surface. The change in surface impedance caused by correlations in a rough-surface sample would then be equal to the change in a diffuse-surface sample, multiplied by the square of the enhancement factor.

On the other hand, suppose that the field enhancement never settled down to a constant value. Then the field beneath a rough surface would not be simply related to that beneath a diffuse surface, and we could not utilize the diffuse calculation of the change in surface impedance to infer the change for the rough surface.

Since the actual $f(z)$ does become a constant when $|az| \gg 1$, the question is whether the minimum distance implied here is short enough to be useful – i.e., to allow us to treat $f(z)$ as a constant. Recall that the calculation of Ref. 4 implied that the interaction which gives rise to the peak in the surface impedance occurs over a volume limited by the depth to which the mode (whose presence is intimately related to the peak) can penetrate. This suggests that if λ (the attenuation length for the mode) is long enough so that $|a\lambda| \gg 1$, then $f(z)$ will be a constant over most of the interaction volume, and the simple multiplicative correction can be used. Conversely, if $|a\lambda| < 1$, then $f(z)$ will be varying all through the interaction region and the

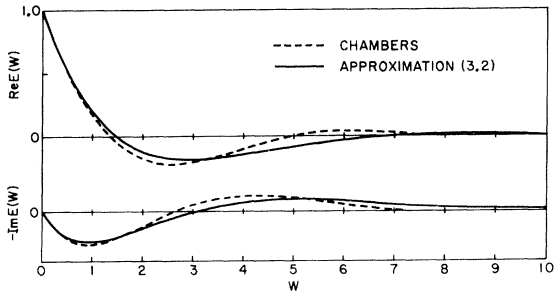

FIG. 4. Real and imaginary parts of the approximate form of the anomalous-skin-effect field at cyclotron resonance used in carrying out these computations. Note that the sign of the imaginary part has been reversed in this figure, purely for display purposes. This reversal also facilitates visual comparison with Chambers's (Ref. 7) presentation, where the opposite sign for the frequency has been used.

simple correction scheme can not be used. This criterion generally excludes the neighborhood of cyclotron resonance from the simple treatment but generally includes the neighborhood of mode threshold – i.e., near $\omega_c/\omega = (1+A_2)^{-1}$.

We have accordingly computed $f^2\Delta Z$, where ΔZ is the change of impedance caused by correlations in a diffuse-surfaced sample, and in Fig. 5, we exhibit the real (resistive) part of this quantity for values of (ω_c/ω) where the simple correction factor might apply. The parameters chosen for Fig. 5 are physically equivalent to those used in Ref. 3, Fig. 7, a figure which was intended to demonstrate that the surface-roughness model did provide a qualitative explanation for the smooth magnetic field dependence of the surface resistance. By using the same parameters here, we demonstrate

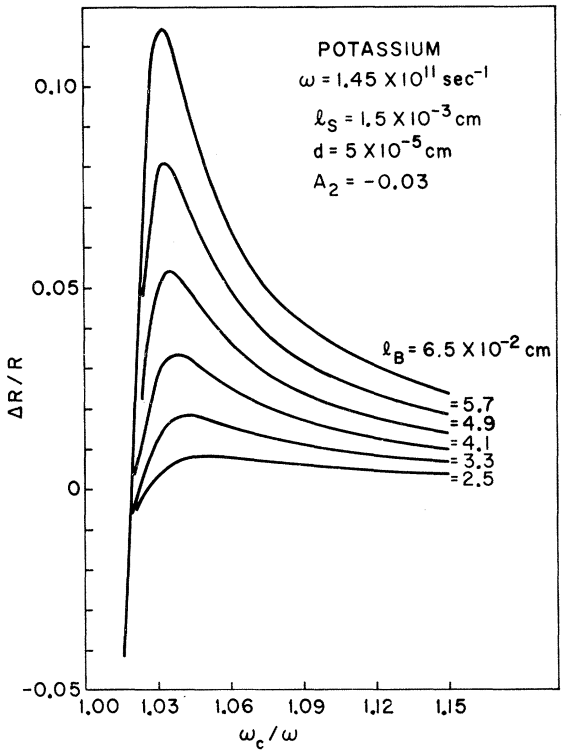

FIG. 5. Change in surface resistance caused by correlations in a rough-surfaced sample of potassium, as a function of magnetic field, for various values of the bulk mean free path. The quantity plotted contains f^2 as a factor, and hence can be changed substantially in size by altering the surface mean free path, as Fig. 1(a) shows. However, the position and shape of the peak depend only on having the phase of f in the rough neighborhood of $-\pi/4$ to $-\pi/3$. Hence, as Fig. 1(b) implies, the position and shape are quite insensitive to the parameters describing the surface, provided only that the surface is sufficiently rough.

that the same model and same parameters also account for the size and position of the correlation-induced peak in the surface resistance – a totally different phenomenon from that responsible for the background variation.

A word of explanation about "physically equivalent parameters" is in order here. The parameters used in Fig. 7, Ref. 3, were $d = 2.5\times10^{-5}$ cm, $l_s = 1.5\times10^{-3}$ cm, and $l_b = 1.5\times10^{-3}$ cm. The text of that article states that l_b, in that context, is to be regarded, not as a true bulk mean free path, but as a mean free path in the lower parts of the skin depth. In effect, one is to think of that calculation as though the model used were

$$l(z) = l_b^0, \qquad z > d^0, \tag{4.1a}$$

$$l(z) = l_b, \qquad d < z < d_0, \tag{4.1b}$$

$$l(z) = (l_b^{-1} + l_s)^{-1}, \qquad 0 < z < d, \tag{4.1c}$$

where l_b^0 is the *true bulk* mean free path and d_0 is some depth on the order of, or greater than, the anomalous-skin depth. The field in this deeper region barely influences the reflection properties, and so the parameters l_b^0 and d_0 have no effect on the surface resistance plotted in Ref. 3.

The problem which arises is to accommodate the model (4.1) into the framework of (3.1). In (4.1), we have a surface mean free path which lies between l_b and $(l_b^{-1}+l_s^{-1})^{-1}$, i.e., between 0.75×10^{-3} cm and 1.5×10^{-3} cm, and a surface thickness which lies between d and d_0. The tendency of the enhancement factor f to become independent of surface thickness (for thicknesses greater than a skin depth) has been noted, and so the choice of thickness is not critical. Our choice for Fig. 5, here, puts the thickness in this noncritical range. For the effective mean free path l_s, we have taken $l_s = 1.5\times10^{-3}$ cm, at one end of the range above. Setting l_s at the other end of the range doubles the size of the peak, but has little effect on its position. Finally, since the bulk mean free path does play an important role in determining the size of the peak, we have varied l_b in Fig. 5 to exhibit this dependence. The bulk mean free path is temperture dependent, and so the various curves in Fig. 5 allow one to deduce how the observed correlation-induced peak will be suppressed as the temperature of the sample rises. The main point is that the same surface-roughness parameters which were used to explain the background variation of surface resistance *do* give sufficient field enhancement to explain the intensity of the correlation-induced peak. These same parameters give a surface-resistance peak very close to $\omega_c/\omega = (1+A_2)^{-1}$. This bears out the conjecture made in Ref. 4 concerning the probable value of the phase of the enhancement factor. In short, the magnitude and phase of the enhance-

ment factor, as calculated using the parameters determined by the over-all surface-resistance calculation of Ref. 3, are such as to account for the size and shape of the correlation-produced peak. Comparing this peak with the experimental peak observed at $\omega_c/\omega = 1.025$[1] gives the value $A_2 = -0.03$, in agreement with the value deduced by Platzman, Walsh, and Foo.[9]

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Influence of the Phonon Spectra of In-Tl Alloys on the Superconducting Transition Temperatures

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A tunneling investigation of the effective phonon density of states through the four distinct crystal structures of the In-Tl alloy series has been performed. The resultant $\alpha^2(\omega)F(\omega)$'s extracted from these studies reflect the characteristics of each individual structure. The data obtained from these experiments serve as a test of existing theoretical expressions for determining T_c of a superconductor from normal-state parameters. It is found that the agreement is somewhat disappointing. In addition, a study of the phonon distribution, or more explicitly, an average phonon energy $\langle\omega\rangle$ in the region of the face-centered cubic to face-centered tetragonal phase transition, suggests an instability in at least one of the phonon modes. This result appears to support the suggestion that this transition is second-order.

I. INTRODUCTION

The ability to predict from various easily determined normal-state parameters whether a material will superconduct and at what temperature has been an elusive dream for many years. Until recently, the only method displaying any success was based on the empirical rules of Matthias,[1] in which, from a previous knowledge of T_c for a wide variety of superconducting elements, compounds, and alloys, and with some experience, one could determine in most cases a considerably better than order-of-magnitude estimate of T_c for a new material. Although this method has survived successfully through several eras in the development of our understanding of superconductivity, it is by no means an accurate determination of T_c. Unfortunately, it gives most people little feel for what physical parameters of the material are the important ones in determining superconductivity. The trends and correlations only hint at the microscopic mechanism or mechanisms responsible for the phenomenon, whereas a detailed study of the pertinent interactions should, in principle, predict accurately the value of T_c.

The BCS theory of superconductivity,[2] suggesting the electron-phonon interaction as the dominant mechanism, predicted that the transition temperature should be given by

$$k_B T_c = 1.14 \langle\omega\rangle e^{-1/N(0)V} ,$$

where $\langle\omega\rangle$ is an average phonon energy, $N(0)$ is the